Undirected & Directed Graphs

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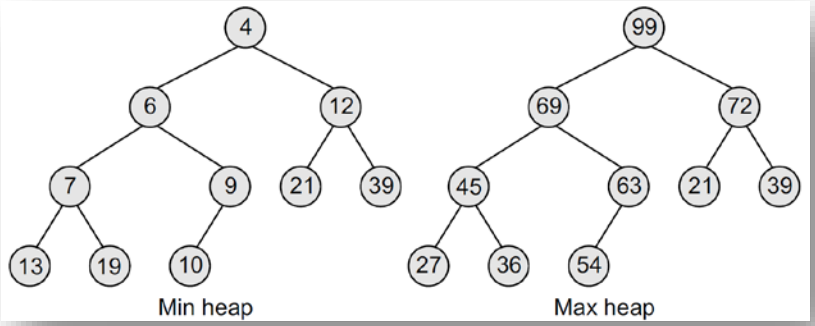
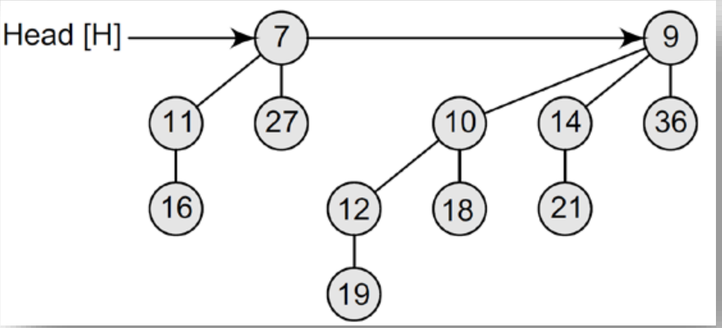
Review

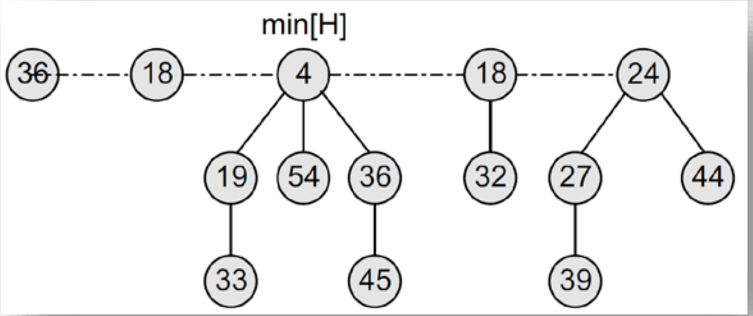
- A **binary heap** is a complete binary tree in which every node satisfies the heap property
  - Min Heap
  - Max Heap



- A **binomial heap** *H* is a set of **binomial trees**
  - Every binomial tree in *H* satisfies the minimum heap property



- A **Fibonacci heap**, which is more flexible then binomial heap, is a collection of trees



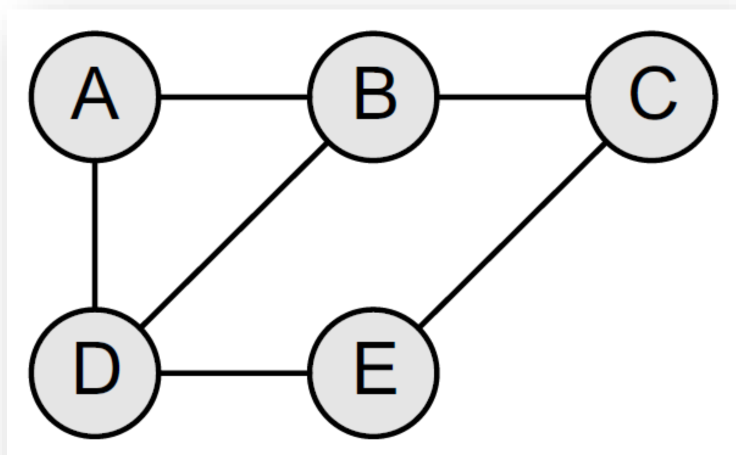
Introduction

- A graph is basically a collection of **vertices** (also called nodes) and **edges** that connect these vertices
  - It is often viewed as a generalization of the tree structure, where instead of having a purely parent-to-child relationship between tree nodes, any kind of complex relationship can exist

- Graphs are widely used to model any situation where entities or things are related to each other in pairs
  - ***Family trees*** in which the member nodes have an edge from parent to each of their children
  - ***Transportation networks*** in which nodes are airports, intersections, or ports

Undirected Graphs

- A graph G is defined as an ordered set (V, E), where $V(G)$ represents the set of vertices and $E(G)$ represents the edges
  - For a given undirected graph with $V(G) = \{A, B, C, D, E\}$ and $E(G) = \{(A, B), (B, C), (A, D), (B, D), (D, E), (C, E)\}$
    - Five vertices or nodes and six edges in the graph



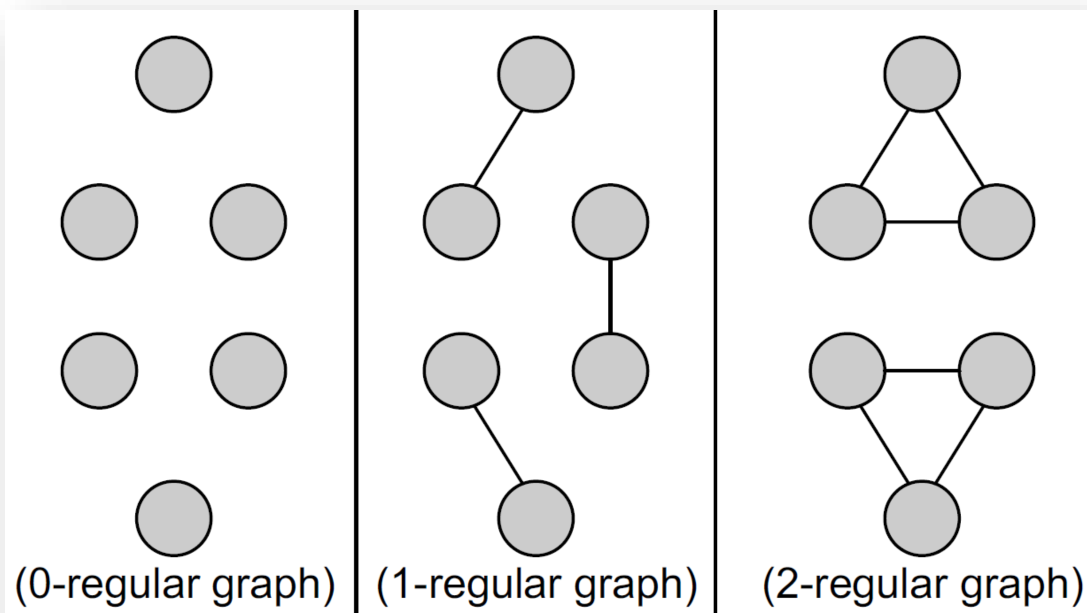
Terminologies for Undirected Graph.

- **Adjacent nodes or neighbors**
  - For every edge, $e = (u, v)$ that connects nodes u and v, the nodes u and v are the end-points and called the **adjacent nodes** or **neighbors**

- **Degree of a node**
  - Degree of a node u, $\deg(u)$, is the total number of edges containing the node u
    - If $\deg(u) = 0$, the node is known as an **isolated node**

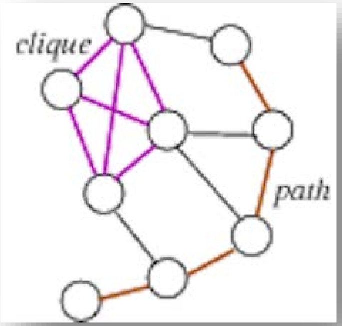
Terminologies for Undirected Graph..

- **Regular graph**
  - It is a graph where each vertex has the same number of neighbors
    - Every node has the same degree
  - A regular graph with vertices of degree k is called a k–regular graph or a regular graph of degree k

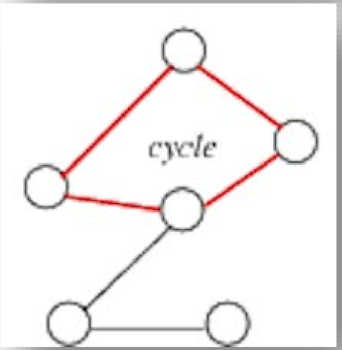


Terminologies for Undirected Graph…

- **Path**
  - A path P written as $P = \{p_0, p_1, p_2, \ldots, p_n)$, of length n from a node u to v is defined as a sequence of $(n+1)$ nodes
    - $p_0 = u$ and $p_n = v$
    - If $u = v$, the path is named **closed path**



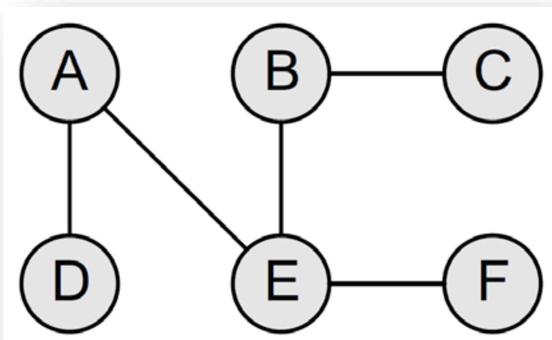
- **Simple path**
  - If all the nodes in the path are distinct
    - An exception is that v_0 can be equal to v_n, which is named **closed simple path**

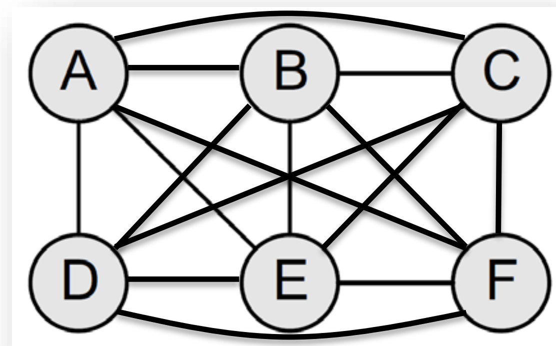


- **Cycle**
  - A path in which the first and the last vertices are same
    - A **simple cycle** has no repeated edges or vertices (except the first and last vertices)
    - Cycle = closed path, simple cycle = closed simple path

Terminologies for Undirected Graph….

- **Connected graph**
  - A graph is said to be connected if for any two vertices (u, v) in V there is a path from u to v
    - There are no isolated nodes in a connected graph
    - A connected graph that does not have any cycle is called a tree

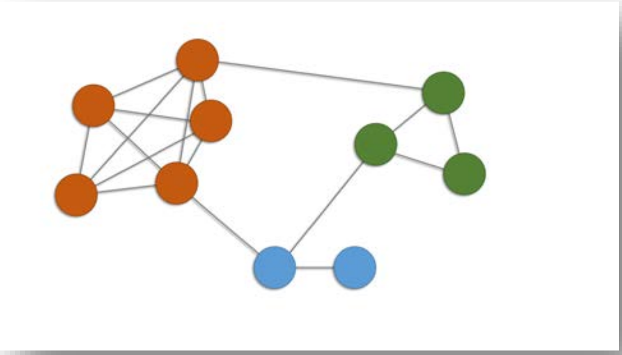




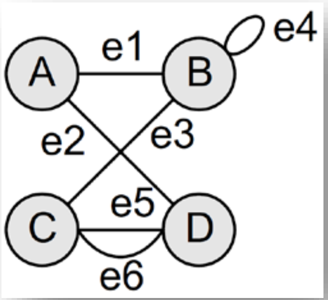
- **Complete graph**
  - If all its nodes are fully connected
  - A complete graph has $\frac{n(n-1)}{2}$ edges

Terminologies for Undirected Graph…..

- **Clique**
  - In an undirected graph $G = (V, E)$, clique is a subset of the vertex set $C \subseteq V$, such that for every two vertices in C, there is an edge that connects two vertices



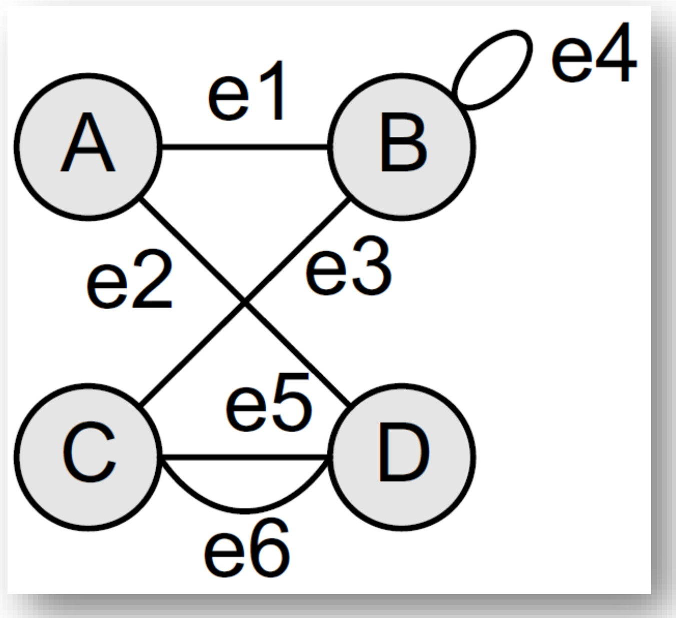
- **Loop**
  - An edge that has identical end-points is called a loop
    - $e = (u, u)$



- **Multiple edges**
  - Distinct edges which connect the same end-points are called multiple edges
    - A graph contains $e = (u, v)$ and $e' = (u, v)$

Terminologies for Undirected Graph……

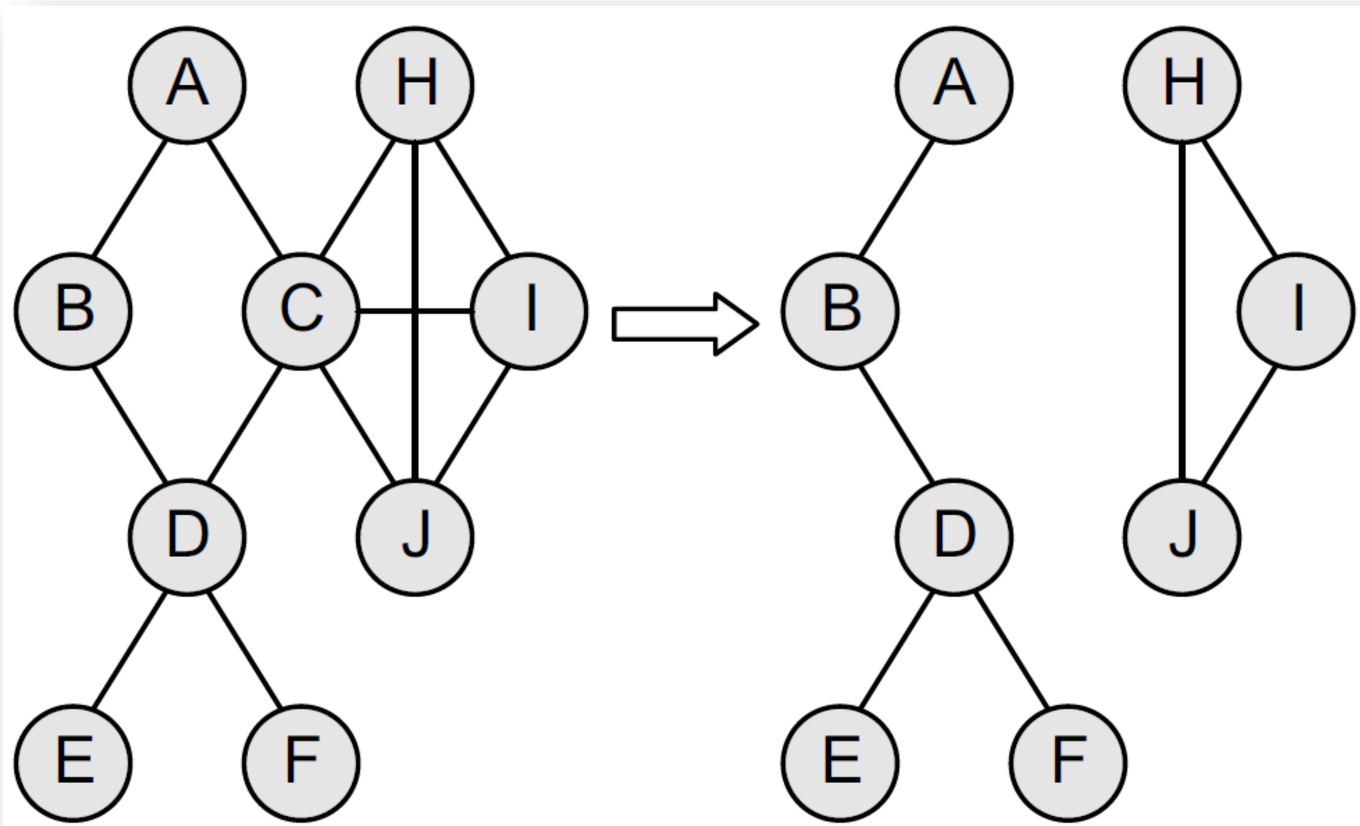
- **Multi-graph**
  - A graph with multiple edges and/or loops is called a multi-graph



- **Size of a graph**
  - The size of a graph is the **total number of edges** in it

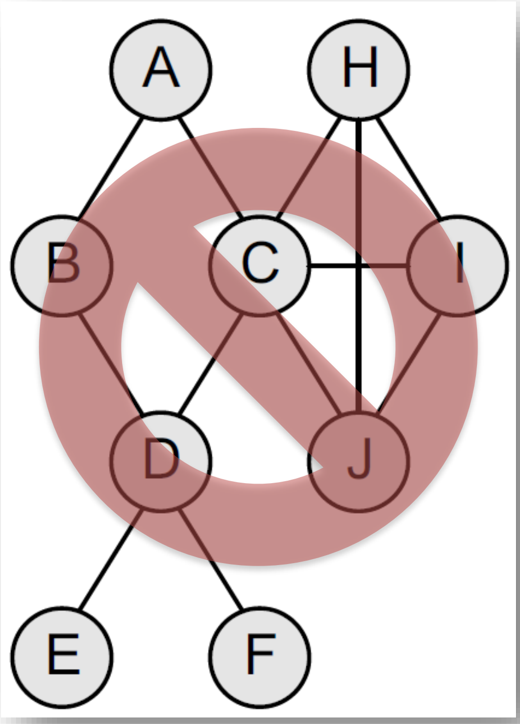
Articulation Point

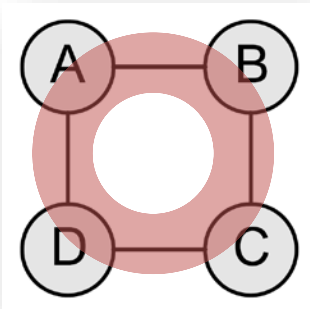
- A vertex v of G is called an articulation point, if removing v along with the edges incident on v, results in a graph that has at least two connected graphs (components)



Bi-connected Graph

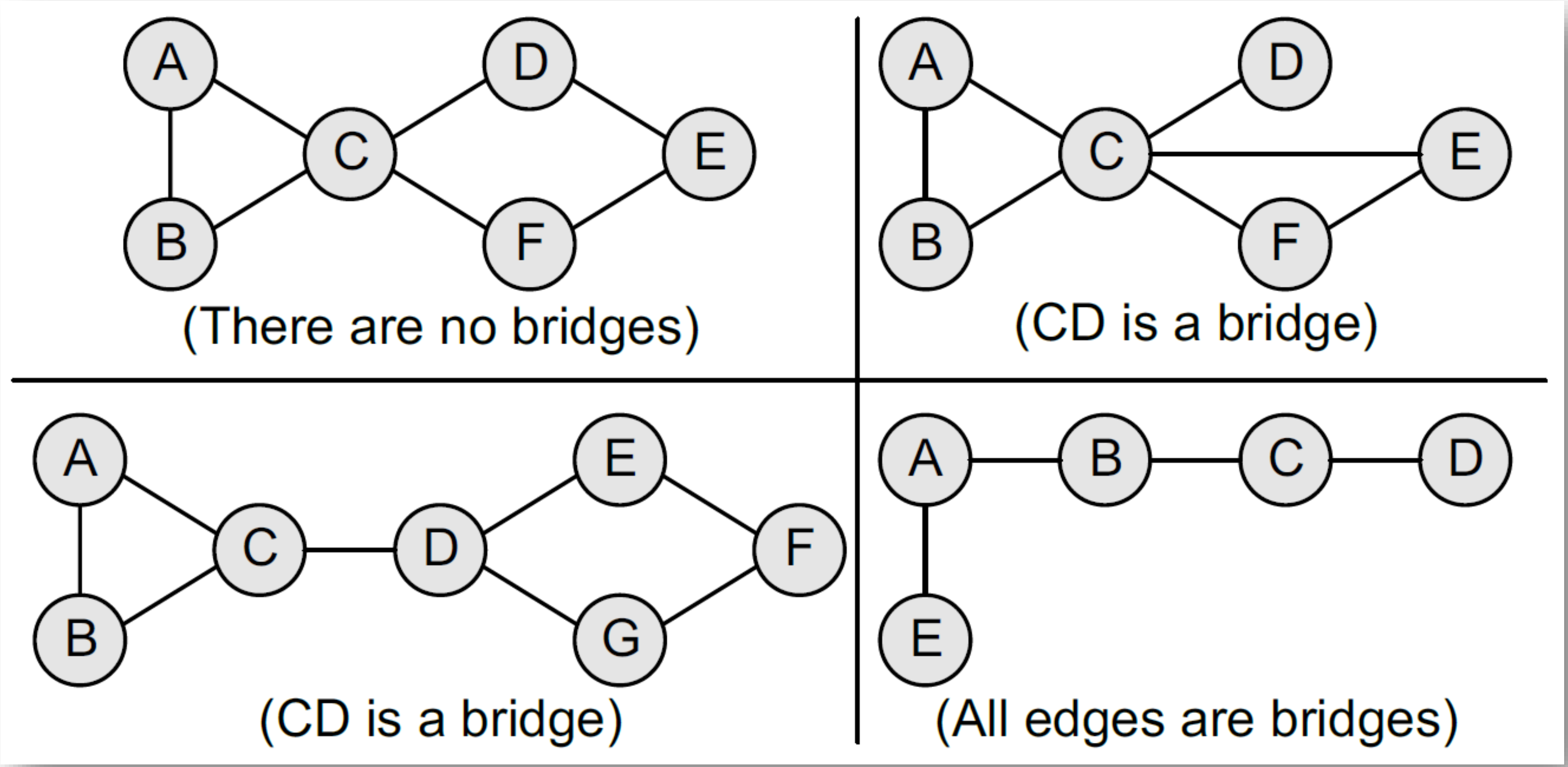
- A **bi-connected graph** is defined as a connected graph that has no articulation vertices
  - In other words, a bi-connected graph is connected and non-separable in the sense that even if we remove any vertex from the graph, the resultant graph is still connected





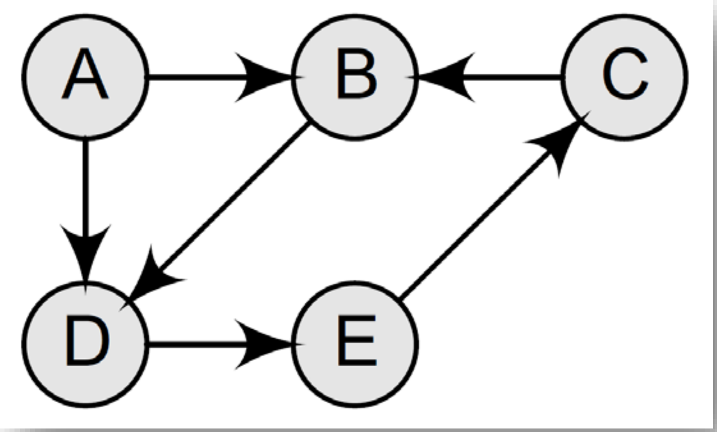
Bridge

- An edge in a graph is called a **bridge** if removing that edge results in a disconnected graph



Directed Graphs

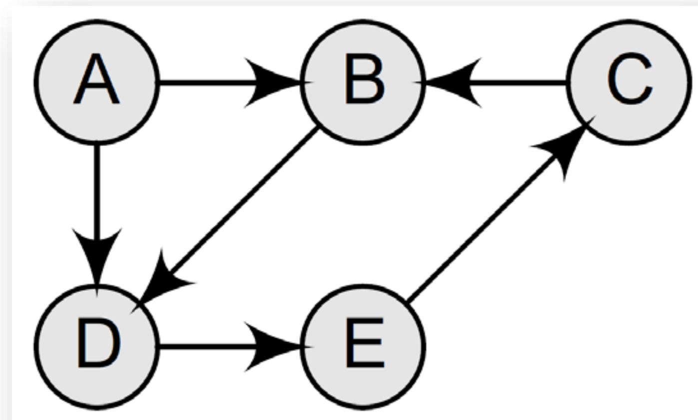
- A graph G is defined as an ordered set (V, E), where $V(G)$ represents the set of vertices and $E(G)$ represents the edges
  - For a given directed graph with $V(G) = \{A, B, C, D, E\}$ and $E(G) = \{(A,B), (C,B), (A,D), (B,D), (D,E), (E,C)\}$
    - Five vertices or nodes and six edges in the graph
  - For a given directed graph, the edge (A, B) is said to initiate from node A (also known as **initial node**) and terminate at node B (**terminal node**)
  - Directed graph is also called **digraph**



Terminologies for Directed Graph.

- **Out-degree of a node**
  - The out-degree of a node u, written as $outdeg(u)$, is the number of edges that originate at u

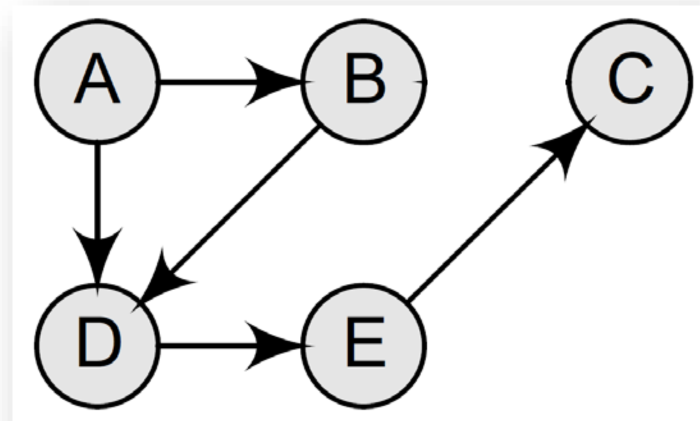
- **In-degree of a node**
  - The in-degree of a node u, written as $indeg(u)$, is the number of edges that terminate at u



- **Degree of a node**
  - The degree of a node, written as $\deg(u)$, is equal to the sum of in-degree and out-degree of that node
  - $\deg(u) = indeg(u) + outdeg(u)$

Terminologies for Directed Graph..

- **Source**
  - A node u is known as a source if it has a positive out-degree but a zero in-degree

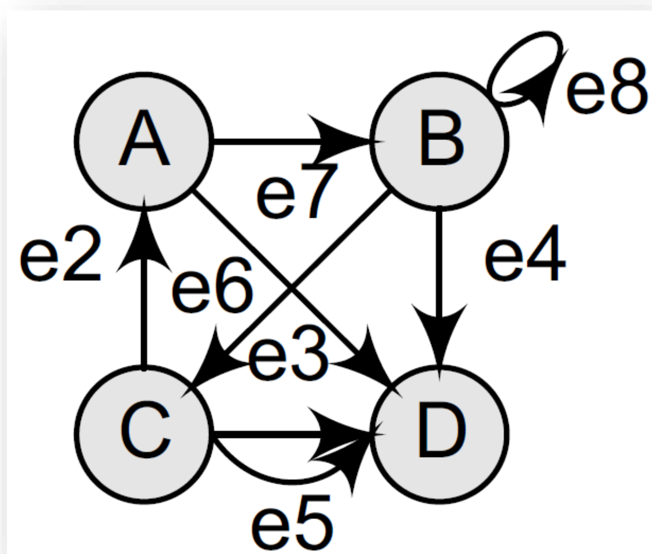


- **Sink**
  - A node u is known as a sink if it has a positive in-degree but a zero out-degree
- **Pendant vertex**
  - A vertex with degree one
    - Also known as leaf vertex

Terminologies for Directed Graph…

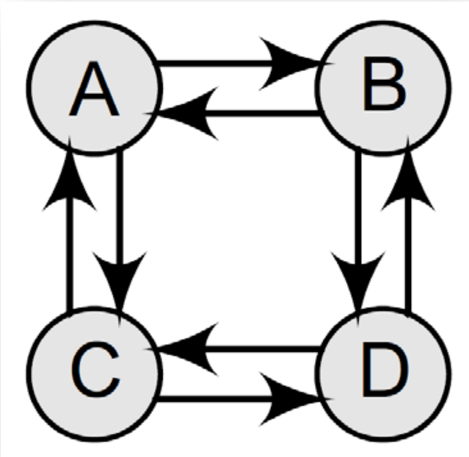
- **Reachability**
  - A node v is said to be reachable from node u, if and only if there exists a (directed) path from node u to node v

- **Parallel/Multiple edges**
  - Distinct edges which connect the same end-points are called multiple edges



Terminologies for Directed Graph….

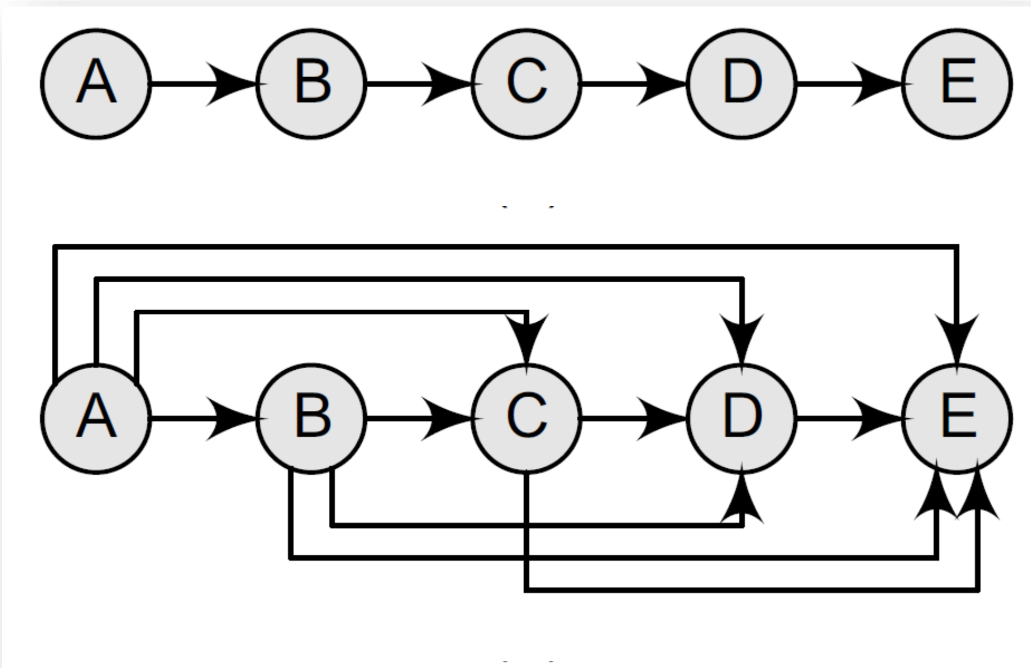
- **Strongly connected directed graph**
  - A digraph is said to be strongly connected if and only if there exists a path between every pair of nodes in G
    - In other words, if there is a path from node u to v, then there must be a path from node v to u



- **Weakly connected digraph**
  - A directed graph is said to be weakly connected if it is connected by ignoring the direction of edges
    - The nodes in a weakly connected directed graph must have either out-degree or in-degree of at least 1

Terminologies for Directed Graph…..

- **Transitive closure**
  - For a directed graph $G = (V, E)$, where V is the set of vertices and E is the set of edges, the transitive closure of G is a graph $G^* = (V, E^*)$
    - In G^*, for every vertex pair v, w in V there is an edge (v, w) in E^* if and only if there is a valid path from v to w in G



Questions?

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